PHYS1112 General Physics I

Solution to Sample Midterm Questions

1. Friction is always along the contact surface → Direction should be either along A or opposite to A.

   m slides down → Friction acting on *m* by *M* is along direction A

   Newton's third law → Friction acting on *M* by *m* is opposite to A

2. $W = 75\mathrm{N} \cdot 42\mathrm{m} = 3150\mathrm{J}$

   $P_{av} = \frac{3150\mathrm{J}}{180\mathrm{s}} = 18\mathrm{W}$

3. Moment arm = Perpendicular distance of the line of action of the force to the axis = 0.10m

   Magnitude of torque = Magnitude of force × Moment arm = 10.0N × 0.10m

   = 1.0 Nm

4. $W = \int_{-2.00\mathrm{m}}^{2.00\mathrm{m}} F dx = S\text{igned area bounded by the curve and } x \text{ axis}$

   $$= 1.00\ \mathrm{Nm} + (-2.00)\mathrm{Nm} = -1.00\mathrm{J}$$

5. Assume that the balls start at ground level, at which the gravitational PE is set to be zero.

   At the highest points, the balls are instantaneously at rest. But acceleration is *g* downward at any time. → (d) is wrong

   By energy conservation, for ball *A*,

   $$\frac{1}{2}mv^2 + 0 = 0 + mgh \Longrightarrow h = \frac{v^2}{2g}$$

   For ball *B*,

   $$\frac{1}{2}m(2v)^2 + 0 = 0 + mgh \Longrightarrow h = \frac{2v^2}{g}$$

   Only (c) is correct.

6. All numbers in this question are assumed to be exact.

$$J = \int_0^{30} F_x dt = \text{Area of triangle} = \frac{30 \times 1000}{2} = 15000\text{Ns}$$

Since acceleration is always along the direction of initial velocity, speed is maximum at the end.

$$mv_f - mv_i = J$$

$$v_f = v_i + \frac{J}{m} = 75 + \frac{15000}{500} = 105\text{m/s}$$

7. Let the angular speed be ω.

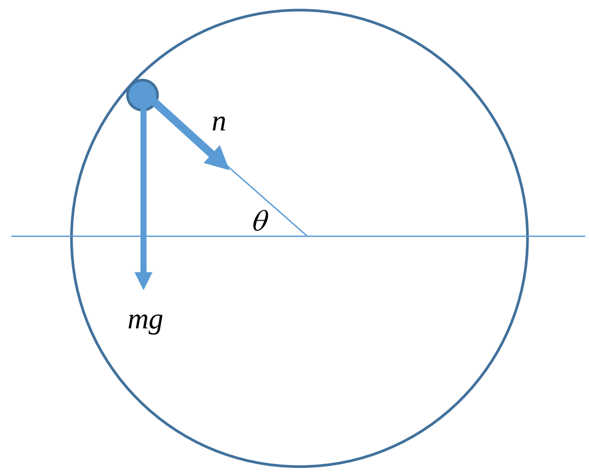
$n + mg\sin\theta = m\omega^2 R$

Lose contact → $n = 0$

$\omega = \sqrt{\frac{g\sin\theta}{R}} = \sqrt{\frac{9.8\sin 68.0°}{0.330}}$

$= 5.25\frac{\text{rad}}{\text{s}} = 5.25\frac{60}{2\pi}\frac{2\pi\text{ rad}}{\text{min}}$

$= 50\text{ rev/min}$



8. Just before released, elastic PE = ½ $k(2x)^2$ = 4× ½ kx^2 → (b) wrong

When it just moves free of the spring, all elastic PE transformed into KE

→KE quadrupled → speed doubled → (a) (c) wrong, (d) correct

When it goes up to maximum height, all KE transformed to gravitational PE

Gravitational PE proportional to height → Maximum height quadrupled → (e) is wrong.

9. By energy conservation

½ m (20.0 m/s)^2 + mg(200m) = ½ mv^2 + mg(160m)

200 (m/s)^2 + 200g m = ½ v^2 + 160g m

$v = \sqrt{2(200 + 40 \times 9.8)}\text{m/s}$

= 34 m/s

10. Let the initial speed be *v*.

By momentum conservation

$0.008\text{kg} \times v = 4.008\text{kg} \times V$

$V = \frac{0.008}{4.008} v$

$\frac{1}{2} \times 4.008\text{kg} \times \left(\frac{0.008}{4.008} v\right)^2 = \frac{1}{2} \times 2400\text{N/m} \times (0.087\text{m})^2$

$v = \sqrt{\frac{2400}{4.008}} \times 0.087 \times \frac{4.008}{0.008}$ =1100m/s

11. By momentum conservation

$1.2\text{kg} \times 0.50 \frac{\text{m}}{\text{s}} = 0.80\text{kg} \times v \cos\theta$

$0.40\text{kg} \times 0.90 \frac{\text{m}}{\text{s}} = 0.80\text{kg} \times v \sin\theta$

Solving the above two equations gives $v^2 = 0.765\ \text{m}^2/\text{s}^2$

Energy released $= \frac{1}{2} \times 0.40 \times 0.90^2 + \frac{1}{2} \times 0.80 \times v^2 - \frac{1}{2} \times 1.2 \times 0.50^2$

$= \frac{1}{2} \times 0.40 \times 0.90^2 + \frac{1}{2} \times 0.80 \times 0.765 - \frac{1}{2} \times 1.2 \times 0.50^2$

$= 0.32\text{J}$

12. Direction of friction on *M* due to *m* should be along A or opposite to A → (c) (d) wrong.

Consider the whole system of *m* + *M*, the only horizontal external force is friction due to the table. Since there is no horizontal acceleration, this force must be zero. → (c) (d) wrong, (e) correct.

13. By energy conservation

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$2 \times 9.8 \times 3 = \frac{1}{2} \times 2 \times v^2 + \frac{1}{2} \times \frac{4 \times 0.25^2}{2} \left(\frac{v}{0.25}\right)^2 = 2v^2$$

$$v = \sqrt{9.8 \times 3} = 5.4\ \text{m/s}$$

14.

$$A = 20.00 \times 4.00 = 80.0\text{N}$$

$$B - \text{A} = 15..00 \times 4.00 \Longrightarrow B = 140.0\text{N}$$

$$C - B = 10..00 \times 4.00 \Longrightarrow C = 180.0\text{N}$$

Smallest tension = A = 80.0N

15. Divide the whole system into the 3 sub-systems of three straight rods.

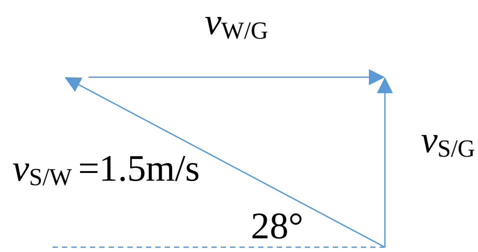
It is obvious that the center of mass of a uniform straight rod is at its midpoint.

Each rod can then be replaced by a point mass of the same mass as the rod at the midpoint (its CM). The system is then reduced to a system of three identical point masses each with mass m, and are located at (0, L/2), (L/2, 0), and (L, L/2).

$$(x_{CM}, y_{CM}) = \frac{m(0, L/2) + m(L/2,0) + m(L, L/2)}{m+m+m} = \frac{1}{3}\left(\frac{3}{2}L, L\right) = \left(\frac{L}{2}, \frac{L}{3}\right)$$

16. W: Water, G: Ground, S: Swimmer

$v_{\text{W/G}} = 1.5\cos 28° = 1.3\text{m/s}$



17. Let the tension of the right string be T and the tension of the left string be T’, and the acceleration of all blocks be a.

Then

$$9.0g - T = 9.0a$$

$$T - T' - 4.0g\sin 30° = 4.0a$$

$$T' - 6.0g\sin 30° = 6.0a$$

From the 1st eq., $T = 9.0(g - a)$

Sub. into 2nd eq. $9.0(g-a) - T' - 4.0g\sin 30° = 4.0a$

$$T' = 7.0g - 13.0a$$

$$a = \frac{7.0g - T'}{13.0}$$

Sub. into the 3rd equation

$$T' - 6.0g\sin 30° = 6.0\frac{7.0g - T'}{13.0}$$

$$\left(1 + \frac{6.0}{13.0}\right)T' = \frac{42.0}{13.0}g + 6.0g\sin 30° = \frac{81.0}{13.0}g$$

$$\frac{19.0}{13.0}T' = \frac{81.0}{13.0}g$$

$$T' = \frac{81.0}{19.0}g = 42\mathrm{N}$$

18. By energy conservation, and the fact that the horizontal velocity is constant,

$$\frac{1}{2}m30.0^2 + 0 = \frac{1}{2}m(30.0\cos 40.0°)^2 + mgh = \frac{1}{2}mv^2 + mg(0.500h)$$

Considering the first term and the third term

$$m30.0^2 = mv^2 + 1.00\,mgh \Longrightarrow mgh = m(900 - v^2)$$

Considering the first term and the second term

$$\frac{1}{2}m30.0^2 = \frac{1}{2}m(30.0\cos 40.0°)^2 + m(900 - v^2)$$

$$450 = 450\cos^2 40.0° + 900 - v^2$$

$$v = \sqrt{450(1 + \cos^2 40.0°)} = 26.7\mathrm{m/s}$$

19. Ignore gravity:

Torque A = (1.0N)(1.0m) = 1.0Nm (Sign: +ve).

For B, the *x*-component points towards the *y* axis and gives no torque.

The torque due to the *z*-component is

Torque B = (2.0N sin30°)(1.0m) = 1.0Nm (Sign: +ve).

Therefore the total torque is 1.0Nm + 1.0Nm = 2.0Nm (Sign: +ve).

Magnitude = 2.0Nm

Including gravity:

Additional torque due to gravitational force (2.00 kg)(9.80ms^{-2}) = 19.6N acting at center of mass.

Additional torque = (19.6N)(0.50m) = 9.8Nm (Sign: -ve)

Total torque = -7.8Nm

Magnitude = 7.8Nm

20. Method 1: Parallel-axes theorem

$$I = \frac{1}{12}ML^2 + M\left(\frac{L}{2} - \frac{L}{3}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{36}ML^2 = \frac{1}{9}ML^2$$

Method 2: From first principle

Let the axis be the *y*-axis.

$$I = \int_{x=-L/3}^{x=2L/3} x^2 dm = \int_{-L/3}^{2L/3} x^2 \frac{M}{L} dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/3}^{2L/3} = \frac{1}{3}\frac{M}{L}\left[\frac{8}{27}L^3 + \frac{1}{27}L^3\right] = \frac{1}{9}ML^2$$

21. Let the final speeds be v_1 and v_2, making angles θ_1 and θ_2 with the initial velocity (counterclockwise angle = +ve)

By momentum conservation

$$mv_i = mv_1 \cos\theta_1 + mv_2 \cos\theta_2 \Longrightarrow v_i = v_1 \cos\theta_1 + v_2 \cos\theta_2$$

$$0 = mv_1 \sin\theta_1 + mv_2 \sin\theta_2 \Longrightarrow 0 = v_1 \sin\theta_1 + v_2 \sin\theta_2$$

By energy conservation

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \Longrightarrow v_i^2 = v_1^2 + v_2^2$$

Squaring the first two equations and add →

$$v_i^2 = v_1^2 + v_2^2 + 2v_1v_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 - \theta_2)$$

Comparing with the third equation: $\cos(\theta_1 - \theta_2) = 0 \Rightarrow |\theta_1 - \theta_2| = \pi/2$

Alternatively, can solve in CM frame, which is moving at $\vec{v}_i/2$.

Observed in CM frame, the total momentum is zero. Hence the two final velocities must be equal but opposite, called $\vec{v}$ and $-\vec{v}$.

By energy conservation in CM frame

$\frac{1}{2}m\left(\frac{v_i}{2}\right)^2 + \frac{1}{2}m\left(\frac{v_i}{2}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 \Longrightarrow 4v^2 = v_i^2$

Now going back to the lab frame, the velocities are $\vec{v} + \vec{v}_i/2$ and $-\vec{v} + \vec{v}_i/2$.

$\left(\vec{v} + \frac{\vec{v}_i}{2}\right) \cdot \left(-\vec{v} + \frac{\vec{v}_i}{2}\right) = -v^2 + \vec{v} \cdot \frac{\vec{v}_i}{2} - \frac{\vec{v}_i}{2} \cdot \vec{v} + \frac{v_i^2}{4} = 0$

Since it is given that the final velocities in lab frame are nonzero, they must be perpendicular.

22. $\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta \Longrightarrow \Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{30^2 - 0^2}{2\times 5.0} = 90\text{rad} = \frac{90}{2\pi}\text{turns} = 14 \text{ turns}$

23.

$$\frac{1}{2}I_{\text{solid}}\omega_{\text{solid}}^2 = \frac{1}{2}I_{\text{shell}}\omega_{\text{shell}}^2$$

$$\frac{\omega_{\text{solid}}}{\omega_{\text{shell}}} = \sqrt{\frac{I_{\text{shell}}}{I_{\text{solid}}}} = \sqrt{\frac{MR^2}{\frac{1}{2}MR^2}} = \sqrt{2} = 1.4$$

24. In the non-inertial frame of the lift, let the downward acceleration of B be *a*’, then A must accelerate to the right with the same acceleration *a*’ (length of string is constant).

In the inertial ground frame, A has a vertically upward acceleration *a* but the horizontal acceleration remains to be *a*’. The downward acceleration of B is

a’ – *a*.

Let the tension of the string be *T*. By Newton’s second law,

T = *ma*’

mg – *T* = *m*(*a*’ – *a*)

So *a*’ = *g* – *T*/*m* + *a*

T = *m* (*g* – *T*/*m* + *a*) = *mg* – *T* + *ma*

2*T* = *m* (*g* + *a*)

T = *m* (*g* + *a*) / 2 = *m* (*g* + *g*/2) / 2 = 3*mg*/4

Supplementary --- Method not taught in this course:

One can still apply Newton's second law to solve the motions in a non-inertial frame accelerating with acceleration $\vec{a}$ provided that one adds to every object m a pseudo force $= -m\vec{a}$.

For an observer in the lift:

$T = ma'$

$mg - T + ma = ma'$

One gets the same equations of motions.

Note that the pseudo force is proportional to mass. This is similar to gravity. In fact in this question with a non-inertial frame under upward acceleration a, all objects are under a downward pseudo force ma in addition to gravity mg. This is equivalent to saying that the non-inertial observer observes a stronger gravity of $g + a$. Therefore, one may first solve for T for the same system on ground as usual, and then just replace g by $g + g/2 = 3g/2$ in the answer.

25. Consider the angular momentum about the rotational axis of the turntable

	Before collision		After collision	
	Angular Momentum	KE	Angular momentum	KE
Turntable	(2.0)(1.5)	½ $(2.0)(1.5)^2$	(2.0)ω	½(2.0) ω^2
Ball	0	½ $(0.40)(3.0)^2$	$[(0.40)(0.80)^2]\,\omega$	½$[(0.40)(0.80)^2]\,\omega^2$

Result: ω = 1.33 rad/s
Initial total KE = 4.05 J
Final total KE = 1.995 J
% energy loss = (4.05 – 1.995) / 4.05 = 51%

26. Sum over vertical forces: $F - Mg = 0$

Torque about an axis through the CM of the yo-yo: $FR = \left(\frac{1}{2}MR^2\right)\alpha$

$\Rightarrow \alpha = 2g/R$